

SYLLABUS

FIRST SEMESTER MATHEMATICS – I

(Lecture hours per week +2 hours of problem working classes)

1. Mathematical Logic

Open sentences compound open sentences - quantifiers - Truth sets - connectives involving quantifiers - methods of proof - methods of disproof. **7 lecture hours.**

2. Relations and Functions

Relations on a set - equivalence relations - equivalence classes - partition of a set. Functions (mappings) - set theoretic properties of function - existence of inverse of a function and properties of inverse functions - composition of functions - associativity - inverse of composition. **8 lecture hours.**

3. Differential Calculus

Successive differentiation - nth derivatives of the functions: $(ax+b)$, $\log(ax+b)$, e^{ax} , $\sin(ax+b)$, $\cos(ax+b)$, $e^{ax} \sin(bx+c)$, $e^{ax} \cos(bx+c)$ - Leibnitz theorem and its applications.

Partial differentiation - first and higher derivatives - Differentiation of homogeneous Functions - Euler's theorem - Total derivative and total differential - Differentiation of implicit functions and composite functions - Jacobians. **8 lecture hours.**

4. Integral Calculus

Reduction formulas for $\int \sin^n x \, dx$, $\int \cos^n x \, dx$, $\int \tan^n x$, $\int \cot^n x \, dx$, $\int \sec^n x \, dx$, $\int \operatorname{cosec}^n x \, dx$, $\int \sin^m x \cos^n x \, dx$. Differentiation under the integral sign. **10 lecture hours.**

5. Analytic Geometry of three - dimensions.

Relation between Cartesian coordinates and position vectors - Distance and Division formulas - Direction cosines of a line (as components of unit vector) - Direction ratios - Angle between two lines - area of a triangle and volume of a tetrahedron with given vertices.

Equation of a line in different forms - perpendicular from a point onto a line. Equation of a plane in different forms - perpendicular from a point onto a plane.

Angle between two planes - Line of intersection of two planes - Plane coaxial with given planes - planes bisecting the angle between two planes- Angle between a line and a plane - Coplanarity of two lines - Shortest distance between two lines.

Equation of the sphere in general and standard forms - equation of a sphere with given ends of a diameter .

Standard equations of quadric surfaces (paraboloid and hyperboloid of one two sheets).

30 Lecture hours.

NOTE: All the derivations (book works) must be through vector methods with reduction to corresponding Cartesian equivalents.

SECOND SEMESTER MATHEMATICS-II

(5 Lecture hours per week + 2 hours of problem working classes)

1. MATRICES

Elementary row and column transformations (operations) - equivalent matrices -finding the inverse of a non-singular matrix by elementary operations. Rank of a matrix - Invariance of rank under elementary transformations - Determination of rank of a matrix by reducing it to the echelon and normal forms.

Homogeneous and non - homogeneous systems of m linear equations in n unknowns - consistency criterion - Solution by elimination method.

Eigen values and eigen vectors of a square matrix - standard properties - Cayley-Hamilton theorem (with proof) - Applications. **20 Lecture hours.**

2. DIFFERENTIAL CALCULUS

Polar coordinates - Angle between the radius vector and the tangent - Angle of intersection of curves (polar form) - polar sub tangent and polar subnormal - perpendicular from pole on the tangent - pedal equations. Derivative of an arc in Cartesian, parametric and polar forms.

Curvature of plane curves - formula for radius of curvature in Cartesian, parametric, polar and pedal forms - center of curvature - evolutes.

Concavity, convexity, Inflexion - singular points - Asymptotes - Envelopes.

Tracing of standard Cartesian, parametric and polar curves (Cissoid, which of Agnesi, Strophoid, Astroid, Folium of Descartes, Catenary, Cycloid, Cardioid, Lemniscate, equiangular Spiral, three leaved rose and four leaved rose) **30 lecture hours.**

3. INTEGRAL CALCULUS

Applications of Integral Calculus: Computation of lengths of arcs, plane areas, and surface area and volume of solids of revolutions for standard curves in Cartesian and polar forms.

10 lecture hours

4. DIFFERENTIAL EQUATIONS

Solutions of ordinary differential equations of first order and first degree:

- i. Variable separable and reducible to variable separable forms.
- ii. Homogeneous and reducible to homogeneous forms.
- iii. Linear equations, Bernoulli equation and those reducible to these.
- iv. Exact equations, equations reducible to exact form with standard integrating factors.

Equations of first order and higher degree - solvable for p , solvable for y - solvable for x - Clairaut's equation - Singular solution-Geometrical meaning.

Orthogonal trajectories in Cartesian and polar forms.

15 lecture hours

THIRD SEMESTER MATHEMATICS-III

(5 Lecture hours per week + 2 hours of problem working classes)

1. GROUP THEORY

Recapitulation of the definition and standard properties of groups and subgroups - Cyclic groups – properties - order of an element of a group - properties related to order of an element - subgroup generated by an element of a group - coset decomposition of a group - moduls relation - index of a group - Lagrange's theorem – consequences. **20 lecture hours**

2. SEQUENCES OF REAL NUMBERS

Definition of a sequence – Bounded sequences - limit of a sequence - convergent, divergent and oscillatory sequences - Monotonic sequences and their properties – Cauchy's criterion. **12 lecture hours**

3. SERIES OF REAL NUMBERS

Definition of convergence, divergence and oscillation of series - properties of convergent series - properties of series of positive terms - Geometric series.

Tests for convergence of series - p-series - comparison tests- Cauchy's root test - D' Alembert's test, Raabe's test - Absolute and conditional convergence - D Alembert test for absolute convergence - Alternating series - Leibnitz test.

Summation of Binomial, Exponential and Logarithmic series. **18 lecture hours**

4. DIFFERENTIAL CALCULUS

Definition of the limit of a function in ϵ - δ form - continuity - types of discontinuities – properties of continuous functions on a closed interval (boundedness, attainment of bounds and taking every value between bounds). Differentiability. Differentiability implies Continuity - converse not true. Rolle's Theorem - Lagrange's and Cauchy's First Mean Value Theorems. Taylor's Theorem (Lagrange's form) - Maclaurin's expansion - Evaluation of limits by L' Hospital's rule. **15 lecture hours**

5. FOURIES SERIES

Trigonometric Fourier series- of functions with period 2π and period $2l$ - Half-range cosine and sine series. **10 lecture hours**

FOURTH SEMESTER MATHEMATICS-IV

(5 Lecture hours per week +2 hours of problem working classes)

1. GROUP THEORY

Normal subgroups - examples and problems - Quotient group - Homomorphism and Isomorphism of groups - Kernel and image of a homomorphism - Normality of the kernel - Fundamental theorem of homomorphism - properties related to isomorphism - permutation group - Cayley's theorem. **15 lecture hours.**

2. DIFFERENTIAL CALCULUS

Continuity and differentiability of functions of two and three variables - Taylor's Theorem and expansions of functions of two variables - Maxima and minima of functions of two variables. Method of Lagrange multipliers. **10 lecture hours.**

3. INTEGRAL CALCULUS

Gamma and Beta functions - results following definitions - Relations connecting the two functions - duplication formula - Applications to evaluation of integrals. **10 lecture hours.**

4. DIFFERENTIAL EQUATIONS

Second and higher order ordinary linear differential equations with constant coefficients - complementary function - Particular integrals (standard types) - Cauchy-Euler differential equation. Simultaneous linear differential equations (two variables) with constant coefficients.

Solutions of second order ordinary linear differential equations with variable coefficients by the following methods.

- i) When a part of complementary function is given
- ii) Changing the independent variable
- iii) Changing the dependent variable
- iv) Variation of parameters
- v) When the equation is exact

Total differential equations - Necessary condition for the equation $Pdx + Qdy + Rdz = 0$ be integrable - Simultaneous equations of the form $dx/P = dy/Q = dz/R$. **18 lecture hours.**

5. LAPLACE TRANSFORMS

Definition and basic properties - Laplace transforms of some common functions and standard results - Laplace transform of periodic functions - Laplace transform of derivatives and the integral of a function - Laplace transforms the Heaviside function and Dirac-delta function - Convolution theorem (No proof) - Inverse Laplace transforms - Laplace transform method of solving ordinary linear differential equations of first and second orders with constant coefficients. **12 lecture hours.**

6. LINER PROGRAMMING

Linear inequalities and their graphs. Statement of the linear programming problem in standard form - classification of solutions - solution of liner programming problem by graphical method.

Illustrative examples on the solution of linear programming problems in two and three variables by the simplex method. **10 lecture hours.**

FIFTH SEMESTER MATHEMATICS-V

(4 Lecture hours per week+ 2 hours of problem working classes)

1. RINGS, INTEGRAL DOMAINS, FIELDS

Rings - Types of rings - properties of rings - Rings of integers modulo n – Subrings - Ideals - principal and Maximal ideals in a commutative ring - examples and standard properties - Homomorphism and isomorphism - properties of homomorphism - Quotient rings - Integral domains - fields - properties following the definition – field is an Integral Domain – Finite Integral Domain is a field. **18 lecture hours.**

2. GEOMETRY OF SPACE CURVES

Vector function of a single scalar variable - its interpretation as a space curve - derivative - tangent, normal and binormal vectors to a space curve - Serret Frenet formulas - simple geometrical applications.

Vector function of two scalar variables - its interpretation as a surface - Tangent plane and Normal to a surface - parametric curves on a surface-parametric curves on the surfaces of a right circular cylinder and sphere - cylindrical spherical polar coordinates.

Orthogonal curvilinear coordinates - Relations between base vector and normal vectors - reciprocity of these vectors - Arc, area and volume elements - specialization to Cartesian, cylindrical and spherical coordinates. **15 lecture hours.**

3. VECTOR DIFFERENTIAL CALCULUS

Scalar field - gradient of a scalar field geometrical meaning - directional derivative - vector field - divergence and curl of a vector field - solenoidal and irrotational fields - scalar and vector potentials - Laplacian of a scalar field - vector identities.

Expressions for $\nabla\phi$, $\text{div } \mathbf{f}$, $\nabla^2\phi$ and $\text{curl } \mathbf{f}$ in orthogonal curvilinear coordinates and specialization to Cartesian, cylindrical and spherical polar coordinates.

15 lecture hours.

4. SPECIAL FUNCTIONS

Polynomial solution of Legendre differential equation - Legendre polynomials - generating function - Recurrence relations - Rodrigue's formula - orthogonality.

Series Solution of Bessel differential equation - Bessel function $J_n(x)$ - Recurrence relation - generating function - orthogonality. **12 lecture hours.**

MATHEMATIC-VI

(4 Lecture hours per week+2 hours of problem working classes)

1. PARTIAL DIFFERENTIAL EQUATIONS

Formation of partial differential equations - Equations of first Order - Lagrange's linear equation - Charpit's method - Standard types of first order non-linear partial differential equations.

Solution of second order linear partial differential equations in two variables with constant coefficients by finding complementary function and particular integral - canonical form for parabolic, elliptic and hyperbolic equations - Solution by separation of variables.

Solutions of one-dimensional heat and wave equations and two-dimensional Laplace equation using Fourier series. **15 lecture hours.**

2. NUMERICAL ANALYSIS

Finite Differences - Definition and properties of Δ , ∇ , δ and E and the relations between them - the n th differences of a polynomial.

Newton-Gregory forward and backward interpolation formulas - Lagrange's and Newton's interpolation formulas for unequal intervals - inverse interpolation.

Numerical differentiation using forward and backward difference formulas - Computation of first and second derivatives.

Numerical Integration: Quadrature formula - Trapezoidal rule - Simpson's 1/3 and 3/8 rules - Weddle's rule. **15 lecture hours.**

3. PARTICLE DYNAMICS

Newton's Laws of motion - Conservative forces and potential energy - Definitions of work, kinetic energy and power.

Motion of a particle in a uniform force field - simple harmonic motion - Two dimensional motion of projectiles.

Tangential and Normal components of velocity and acceleration - Radial and Transverse components of velocity and acceleration - Constrained Motion of a Particle under gravity along inside and outside of a circle.

Motion of a particle in a central force field - Determination of orbit from central forces and vice versa.

Linear momentum, angular momentum and energy of a system of particles - Principles of linear momentum, angular momentum and energy- Motion of centroid and motion relative to centroid. **30 lecture hours.**

SIXTH SEMESTER MATHEMATICS-VII

(4 Lecture hours per week +2 hours of problem working classes)

1. LINEAR ALGEBRA

Vector space – Examples – Properties – Subspaces - criterion for a subset to be a subspace- Linear combination - linear independent and dependent subsets - Basis and dimension - Standard results - Examples illustrating concepts and results.

Linear transformations - properties - matrix of a linear transformation - change of basis - range and kernel - rank and nullity-Rank - Nullity theorem - Non-singular linear transformations.

Eigen values and Eigen vectors of a linear transformation - Interpretation in terms of matrices - Examples illustrating the concepts. **18 lecture hours.**

2. LINE AND MULTIPLE INTEGRALS

Definition of a line integral and basic properties – examples on evaluation of line integrals.

Definition of a double integral - its conversion to iterated integrals - evaluation of double integrals by change of order of integration and by change of variables - Computation of plane and surface areas, volume underneath a surface and volume of evolution using double integrals.

Definition of a triple integral and evaluation - change of variables - volume as a triple integral. **15 lecture hours.**

3. INTEGRAL THEOREMS

Line, surface and volume integrals of vector functions - Green's theorem in the plane (with proof). Direct consequences of the theorem.

The divergence theorem (with proof) - Direct consequences of the theorem.

The Stokes' theorem (with proof) - Direct consequences of the theorem. **15 lecture hours.**

4. CALCULUS OF VARIATIONS

Variation of function $f = f(x, y, y^1)$ – variation of the corresponding functional Extremal of a functional - variational problem - Euler's equation and its particular forms - Examples - standard problems like geodesics, minimal surface of revolution, hanging chain, Brachistochrone problem - Isoperimetric problem. **12 lecture hours.**

MATHEMATICS- VIII

(4 Lecture hours per week + 2 hours of problem working classes)

1. COMPLEX ANALYSIS

Complex numbers - The complex plane conjugate and modulus of a complex number - polar form - geometrical representation – Euler's formula: $e^{i\theta} = \cos \theta + i \sin \theta$

Function of a complex variable: Limit, continuity and differentiability.

Analytic function - Cauchy-Riemann equations in Cartesian and polar forms - Sufficiency conditions for analyticity (in Cartesian form) - Standard properties of analytic functions - Construction of analytic functions, given real or imaginary parts - Milne-Thomson method.

Transformations - definition of a conformal transformation. Examples.

Discussion of the transformations: $w = z^2$, $w = \sin z$, $w = \cos z$, $w = e^z$, $w = \cosh z$, $w = \frac{1}{2} (z + 1/z)$.

The bilinear transformation - cross ratio property - Bilinear transformation - transforms circles into circles or lines – problems - thereon.

The complex line integral - Examples and properties.

Cauchy's Integral theorem (proof using Green's theorem) and its direct consequences. The Cauchy's integral formula for the function and the derivatives. Applications to evaluation of simple line integrals - Cauchy's inequality - Liouville's theorem - Fundamental theorem of algebra.

30 lecture hours.

2. FOURIER TRANSFORMS

The Fourier integral - Complex Fourier transform - inverse transform - Basic properties - Transforms of the derivative and the derivative of the transform - problems thereon.

Fourier sine and cosine transforms and inverses - transforms for first and second order derivatives - problems thereon.

15 lecture hours.

3. NUMERICAL ANALYSIS

Solution of Algebraic and transcendental equations. Method of successive bisection - Method of false position - secant method - Newton-Raphson method.

Numerical solutions of non-homogeneous systems of linear algebraic equations in 3 variables by Jacobi's and Gauss - Seidel methods - Computation of largest eigen value of a square matrix by power method.

Solution of initial value problems for ordinary linear first order. Differential equations by Taylor's, Euler's, Modified Euler's, Runge-Kutta method.

15 lecture hours.